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On (i, j)-Ipre*open sets in Bi-Intuitionistic Topological Spaces

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Abstract: **D. Coker introduced the concept of intuitionistic sets. A tuple (X, T_i, T_j), whereas $X \neq \emptyset$ as well as T_i, T_j are topologies on X with $i \neq j$ is called a bi-topological space introduced by Kelly in 1963. In 2015, Taha H. Jassim et al. defined (i, j)- Ipre, (i, j)- Isemi, (i, j)- Iα, (i, j)- Iβ open sets in bi-intuitionistic topological spaces. In this paper, we introduce a new class of (i, j)-Ipre*-open sets in bi-Intuitionistic Topological Spaces. Based on this concept, we define several types of open sets, namely (i, j)-Isemi*-open, (i, j)-Iα^*-open, (i, j)-Iβ^*-open sets in bi-intuitionistic topological spaces.**

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1. INTRODUCTION

The theory of fuzzy sets was first introduced by Zadeh in 1965 to deals with uncertainty and vagueness in mathematical modelling. In 1996 coker [1] introduced the concept of intuitionistic sets. A triple (X, T_i, T_j), where $X \neq \emptyset$ and T_i, T_j are topologies on X with $i \neq j$ is called a bi-topological space introduced by Kelly [4] in 1963. Combining the ideas of intuitionistic topology and bi-topology, Taha H. Jassim et al. developed the concepts of (i, j)- Ipre open set, (i, j)- semi open set, (i, j)- Iα open set and (i, j)- Iβ open sets in bi-intuitionistic topological spaces in 2015. In this paper, we introduce the notion of (i, j)-Intuitionistic pre* open sets in bi-intuitionistic topological spaces and investigate their fundamental properties.

2. PRELIMINARIES

Definition 2.1 [2] Let X be a nonempty fixed set. An intuitionistic set (IS for short) $\Re$ is an object having the form $\Re = \langle X, \Re_1, \Re_2 \rangle$ where $\Re_1, \Re_2$ are subsets of X satisfying $\Re_1 \cap \Re_2 = \emptyset$. The set $\Re_1$ is called the set of members of $\Re$, while $\Re_2$ is called set of non-members of $\Re$.

Definition 2.2 [2] Let $\Re$ and $\mathfrak{K}$ be two intuitionistic sets having the form $\Re = \langle X, \Re_1, \Re_2 \rangle$ and $K = \langle X, K_1, K_2 \rangle$ respectively

a) $\Re \subseteq K \Leftrightarrow \Re_1 \subseteq K_1 \,\&\, \Re_2 \supseteq K_2$

b) $\Re = K \Leftrightarrow \Re \subseteq K \,\&\, K \subseteq \Re$

c) $\overline{\Re} = \langle X, \Re_2, \Re_1 \rangle$

d) $\Re \cap K = \langle X, \Re_1 \cap K_1, \Re_2 \cup K_2 \rangle$

e) $\Re \cup K = \langle X, \Re_1 \cup K_1, \Re_2 \cap K_2 \rangle$

f) $X = \langle X, X, \emptyset \rangle$

g) $\emptyset = \langle X, \emptyset, X \rangle$

Definition 2.3 [2] An intuitionistic topology (IT) on a nonempty set X is a family T of IS's in X containing $\emptyset$, X and closed under finite intersection and arbitrary union. In this case the pair (X, IT) is called an **intuitionistic topological space** (ITS) and any IS in IT is known as an intuitionistic open set (IOS) in X, the complement of IOS is called intuitionistic closed set (ICS) in X.

Definition 2.4 [2] If (X, IT) be ITS, and let $\mathfrak{R} = \langle X, \mathfrak{R}_1, \mathfrak{R}_2 \rangle$ be IS in X then the intuitionistic interior of $\mathfrak{R}$ (shortly, Iint ($\mathfrak{R}$)) and intuitionistic closure of $\mathfrak{R}$ (shortly, Icl($\mathfrak{R}$)) are defined by

a) $\text{Iint}(\mathfrak{R}) = \cup \{G: G \text{ is an IOS and } G \subseteq \mathfrak{R}\}$

b) $\text{Icl}(\mathfrak{R}) = \cap \{F: F \text{ is an ICS and } \mathfrak{R} \subseteq F\}$

Definition 2.5 [3] If (X, IT) is an ITS and an intuitionistic set $\mathfrak{R}$ is known as the Ig-CS if $\text{Icl} \subseteq \mathfrak{R}$ whenever $\mathfrak{R} \subseteq U$ and U is IOS in X. The complement of Ig-CS is known as the Ig-OS.

If (X, IT) is an ITS and $\mathfrak{R}$ be an IS in X then the definition of Ig-closure of $\mathfrak{R}$ is, $\text{Icl}^*(\mathfrak{R}) = \cap \{\mathfrak{H}: \mathfrak{H} \text{ is Ig-CS in X } \& \mathfrak{R} \subseteq \mathfrak{H}\}$.

Theorem 2.6 [2] If (X, IT) is an ITS and $\mathfrak{R}$ and K are IS in X then the followings are hold.

a) $\text{Iint}(\emptyset) = \emptyset$ and $\text{Iint}(X) = X$

b) $\mathfrak{R}$ is IOS iff $\mathfrak{R} = \text{Iint}(\mathfrak{R})$

c) $\mathfrak{R} \subseteq K \Rightarrow \text{Iint}(\mathfrak{R}) \subseteq \text{Iint}(K)$

d) $\text{Iint}(\mathfrak{R} \cap K) = \text{Iint}(\mathfrak{R}) \cap \text{Iint}(K)$

e) $\text{Iint}(\mathfrak{R} \cup K) \supseteq \text{Iint}(\mathfrak{R}) \cup \text{Iint}(K)$

f) $\text{Icl}(\emptyset) = \emptyset$ and $\text{Icl}(X) = X$

g) $\mathfrak{R}$ is ICS iff $\mathfrak{R} = \text{Icl}(\mathfrak{R})$

h) $\mathfrak{R} \subseteq K \Rightarrow \text{Icl}(\mathfrak{R}) \subseteq \text{Icl}(K)$

i) $\text{Icl}(\mathfrak{R} \cap K) \subseteq \text{Icl}(\mathfrak{R}) \cap \text{Icl}(K)$

j) $\text{Icl}(\mathfrak{R} \cup K) = \text{Icl}(\mathfrak{R}) \cup \text{Icl}(K)$

Theorem 2.7 [3] Let (X, IT) is an ITS and $\mathfrak{R}$, $\mathfrak{H}$ are IS's in X then the followings are hold.

a) $\text{Iint}^*(X) = X$ and $\text{Iint}^*(\emptyset) = \emptyset$

b) If $\mathfrak{R}$ is Ig-open set then $\mathfrak{R} = \text{Iint}^*(\mathfrak{R})$

c) $\text{Iint}(\mathfrak{R}) \subseteq \text{Iint}^*(\mathfrak{R}) \subseteq \mathfrak{R}$

d) If $\mathfrak{R} \subseteq \mathfrak{H}$ then $\text{Iint}^*(\mathfrak{R}) \subseteq \text{Iint}^*(\mathfrak{H})$

e) $\text{Iint}^*(\mathfrak{R} \cap \mathfrak{H}) \subseteq \text{Iint}^*(\mathfrak{R}) \cap \text{Iint}^*(\mathfrak{H})$

f) $\text{Icl}^*(X) = X$ and $\text{Icl}^*(\emptyset) = \emptyset$

g) If $\mathfrak{R}$ is Ig-closed set then $\mathfrak{R} = \text{Icl}^*(\mathfrak{R})$

h) $\mathfrak{R} \subseteq \text{Icl}^*(\mathfrak{R}) \subseteq \text{Icl}(\mathfrak{R})$

i) If $\mathfrak{R} \subseteq \mathfrak{H}$ then $\text{Icl}^*(\mathfrak{R}) \subseteq \text{Icl}^*(\mathfrak{H})$

j) $\text{Icl}^*(\mathfrak{R} \cap \mathfrak{H}) \subseteq \text{Icl}^*(\mathfrak{R}) \cap \text{Icl}^*(\mathfrak{H})$

Definition 2.8 [5] Let (X, IT_i, IT_j) be a **bi-intuitionistic topological space**(bi-ITS) if for each of (X, IT_i) and (X, IT_j) are ITS on X and let G ⊆ X then G is said to be (i, j)-intuitionistic open set ((i, j)-IOS for short) if G = $\Re \cup \Im$ where $\Re \in IT_i$ and $\Im \in IT_j$. The complement of an (i, j)-intuitionistic open set is called an (i, j)-intuitionistic closed set ((i, j)-ICS for short).

Example 2.9 [5] Let X = {1, 2, 3} and IT_i = {∅, X, A, B, C} where A = <X, {3}, {1, 2}>, B = <X, {1}, {3}>, C = <X, {1, 3}, ∅> and IT_j = {∅, X, D, E} where D = <X, {1}, {2}>, E = <X, {1}, {2, 3}>. (i, j)-IOS = {∅, X, A, B, C, D, E, F, G} where F = <X, {1, 3}, {2}>, G = <X, {1}, ∅>. (i, j)-ICS = {∅, X, $A^c, B^c, C^c, D^c, E^c, F^c, G^c$}

Definition 2.10 [5] Let (X, IT_i, IT_j) be a bi-ITS and $\Re$ = <X, $\Re_1$, $\Re_2$> is intuitionistic set in X, then the intuitionistic interior and intuitionistic closure of $\Re$ are denoted by (i, j)-Iint($\Re$) and (i, j)-Icl($\Re$) respectively and defined as a union of all (i, j)-IOS of X that contained in $\Re$ and the intersection of all (i, j)- ICS in X that contain $\Re$ respectively.

Definition 2.11 [5] Let (X, IT_i, IT_j) bi-ITS an intuitionistic set $\Re$ = <X, $\Re_1$, $\Re_2$> in X is said to be an (i, j)-intuitionistic generalized closed set (shortly, (i, j)-Ig-CS), if $IT_j cl(\Re) \subseteq U$, whenever $\Re \subseteq U$ and U is an IT_i-IOS. The complement of an (i, j)-Ig-CS is called an (i, j)- intuitionistic generalized open set (shortly, (i, j)-Ig-OS).

Definition 2.12 [5] Let (X, IT_i, IT_j) be a bi-ITS and let $\Re$ = <X, $\Re_1$, $\Re_2$> be IS in X. Then $\Re$ is called:

a) (i, j) intuitionistic semi-open set ((i, j)-ISOS, for short) if $\Re \subseteq IT_j cl\,(IT_i int(\Re))$

b) (i, j) intuitionistic α-open set ((i, j)-IαOS, for short) if $\Re \subseteq IT_i int\,(IT_j cl\,(IT_i int(\Re)))$

c) (i, j) intuitionistic pre-open set ((i, j)-IPOS, for short) if $\Re \subseteq IT_i int\,(IT_j cl(\Re))$

d) (i, j) intuitionistic β-open set ((i, j)-IβOS, for short) if $\Re \subseteq IT_j cl\,(IT_i int\,(IT_j cl(\Re)))$

The complement of (i, j)-ISOS (resp. (i, j)-IαOS, (i, j)-IPOS and (i, j)-IβOS) is called (i, j) intuitionistic semi-closed set (resp. (i, j) intuitionistic α-closed set, (i, j) intuitionistic pre-closed set and (i, j) intuitionistic β-closed set in X. ((i, j)-ISCS, (i, j)-IαCS, (i, j)-IPCS, (i, j)-IβCS, for short).

3. ON (i, j)- IPRE* OPEN SETS

We say that (X, IT_i, IT_j), bi-intuitionistic topological space if for each of (X, IT_i) and (X, IT_j) is intuitionistic topological space on X. In this section, we introduce the concept of (i, j)- Ipre*open sets and investigate several related properties and results.

Definition 3.1 Let bi-ITS (X, IT_i, IT_j), $\Re \subseteq$ X, then $\Re$ is said to be **(i, j)-intuitionistic pre*open set** (shortly, an **(i, j)-Ipre*open set**) if $\Re \subseteq IT_i int(IT_j cl^*(\Re))$. The family of all (i, j)-Ipre*open sets in X is notated by (i, j)-IP*O (X, IT_i, IT_j).

Example 3.2 Let X = {a, b, c}, IT_i = {X, ∅, <X, {a}, ∅>, <X, {a}, {b}>, <X, {a, b}, ∅>, <X, ∅, {a, c}>, <X, {a, c}, ∅>} and IT_j = {X, ∅, <X, ∅, ∅>, <X, {a}, ∅>, <X, ∅, {b}>, <X, {c}, ∅>, <X, {a}, {b}>, <X, {a, c}, ∅>}. Here, (i, j)-Ipre*open = {X, ∅, <X, {a}, ∅>, <X, {a}, {b}>, <X, {a, b}, ∅>, <X, ∅, {a, c}>, <X, {a, c}, ∅>, <X, {a, c}, {b}>}. If we have $\Re$ = <X, {a}, ∅>, then $IT_j cl^*$(<X, {a}, ∅>) = <X, {a, b}, ∅>, $IT_i int$(<X, {a, b}, ∅>) = <X, {a, b}, ∅>. Therefore, <X, {a}, ∅> ⊆ <X, {a, b}, ∅>. Hence <X, {a, b}, ∅> is (i, j)-IP*O (X, IT_i, IT_j). In general, X and ∅ are (i, j)-IP*O (X, IT_i, IT_j).

Definition 3.3 Let (X, IT_i, IT_j) be bi-ITS, $\Re$ is said to be (j, i)-intuitionistic pre* open set (shortly, an (j, i)-Ipre*open set) if $\Re \subseteq IT_j int(IT_i cl^*(\Re))$. s

Example 3.4 consider example 3.2, (j, i)-Ipre*open set = {X, ∅, <X, ∅, ∅>, <X, {a}, ∅>, <X, ∅, {b}>, <X, ∅, {c}>, <X, {c}, ∅>, <X, {a}, {b}>, <X, {a}, {c}>, <X, {a, b}, ∅>, <X, ∅, {b, c}>, <X, {a, c}, ∅>, <X, {a}, {b, c}>, <X, {a, b}, {c}>}. IT_i-pre* open set = {X, ∅, <X, {a}, ∅>, <X, {a}, {b}>, <X, {a}, {c}>, <X, {a, b}, ∅>, <X, ∅, {a, c}>, <X, {a, c}, ∅>, <X, {a}, {b, c}>, <X, {a, b}, {c}>}. IT_j-pre* open set = {X, ∅, <X, ∅, ∅>, <X, {a}, ∅>, <X, ∅, {b}>, <X, {c}, ∅>, <X, {a}, {b}>, <X, {c}, {b}>, <X, {a, c}, ∅>, <X, {a, c}, {b}>}.

Theorem 3.5 In a bi-ITS (X, IT_i, IT_j), each IT_i-open set in X is (i, j)-Ipre*open set and each IT_j-open set in X is (j, i)-Ipre*open set.

Proof: Let $\Re$ be any IT_i-open set in X, then $\Re = IT_i int(\Re)$. Always, $\Re \subseteq IT_j cl^*(\Re) \Rightarrow IT_i int(\Re) \subseteq IT_i int(IT_j cl^*(\Re)) \Rightarrow \Re \subseteq IT_i int(IT_j cl^*(\Re))$. Thus, $\Re$ is (i, j)-Ipre*open set.

Similarly, each IT_j-open set in X is (j, i)-Ipre*open set.

Remark 3.6 The afterwards example shows that the previous theorem converse doesn't have too accurate.

Example 3.7 Let X = {s, t, u}, IT_i = {X, ∅, <X, {s}, {t, u}>, <X, {t}, {s, u}>, <X, {s, t}, {u}>} and IT_j = {X, ∅, <X, {s}, {t}>, <X, {t}, {u}>, <X, {s, t}, ∅>, <X, ∅, {t, u}>}. Clearly that, <X, {s, t}, ∅> is (i, j)-Ipre*open set but not IT_i-open set and <X, {s}, {t, u}>, <X, {s, t}, {u}> are (j, i)-Ipre*open set but not IT_j-open set.

Theorem 3.8 The union of an arbitrary collection of (i, j)-Ipre*open sets are also an (i, j)-Ipre*open set.

Proof: Suppose $\{\Re_\alpha/\alpha \in I\}$ is a family of (i, j)- Ipre*open set in bi-ITS. Then for each $\alpha \in I$, $\Re_\alpha \subseteq IT_i int(IT_j cl^*(\Re_\alpha))$. Since $\Re_\alpha \subseteq \cup \Re_\alpha$ then $IT_i int(IT_j cl^*(\Re_\alpha)) \subseteq IT_i int(IT_j cl^*(\cup \Re_\alpha))$ and also $\Re_\alpha \subseteq IT_i int(IT_j cl^*(\cup \Re_\alpha))$. Therefore, $\cup \Re_\alpha \subseteq IT_i int(IT_j cl^*(\cup \Re_\alpha))$. Hence $\cup \Re_\alpha$ is (i, j)-Ipre*open set.

Remark 3.9 The intersection of two (i, j)-Ipre*open sets are need not be an (i, j)-Ipre*open set.

Example 3.10 Let X = {s, t}, IT_i = {X, ∅, <X, ∅, ∅>, <X, {s}, ∅>, <X, {t}, ∅>, <X, ∅, {t}>} and IT_j = {X, ∅, <X, ∅, {s}>, <X, {s}, ∅>, <X, {t}, {s}>} then (i, j)-Ipre*open = {X, ∅, <X, ∅, ∅>, <X, {s}, ∅>, <X, {t}, ∅>, <X, ∅, {t}>, <X, {t}, {s}>}. Clearly that, $\Re$ = <X, {s}, ∅> and $\Im$ = <X, {t}, {s}>. Now $\Re \cap \Im$ = <X, ∅, {s}> ∉ (i, j)-IP*O (X, IT_i, IT_j).

Lemma 3.11 Let (X, IT_i, IT_j) be a bi-ITS and let $\Re$ and $\Im$ be any IS's in X. If $\Im$ is an IT_i-open set then $IT_j cl^*(\Re) \cap \Im \subseteq IT_j cl^*(\Re \cap \Im)$.

Theorem 3.12 Let (X, IT_i, IT_j) be a bi-ITS. If $\Re$ is (i, j)-Ipre*open sets and $\Im$ is IT_i-open set then $\Re \cap \Im$ is (i, j)-Ipre*open set.

Proof: Let $\Re$ be (i, j)-Ipre*open sets in X and $\Im$ is IT_i-open set, then $\Re \subseteq IT_i int(IT_j cl^*(\Re))$ & $\Im = IT_i int(\Im)$. Now, $\Re \cap \Im \subseteq IT_i int(IT_j cl^*(\Re)) \cap IT_i int(\Im) = IT_i int(IT_j cl^*(\Re) \cap \Im) \subseteq IT_i int(IT_j cl^*(\Re \cap \Im))$ by lemma 3.11. Thus, $\Re \cap \Im$ is (i, j)-Ipre*open set.

Theorem 3.13 In a bi-ITS (X, IT_i, IT_j), each (i, j)-Ipre*open set is (i, j)-Ipre open set.

Proof: Let $\Re$ be (i, j)-Ipre*open set then $\Re \subseteq IT_i int(IT_j cl^*(\Re))$. By preliminaries 2.7 (h), $IT_j cl^*(\Re) \subseteq IT_j cl(\Re) \Rightarrow IT_i int(IT_j cl^*(\Re)) \subseteq IT_i int(IT_j cl(\Re)) \Rightarrow \Re \subseteq IT_i int(IT_j cl(\Re))$. Thus, $\Re$ is (i, j)-Ipre open set.

Remark 3.14 The afterwards example shows that the previous theorem converse doesn't have too accurate.

Example 3.15 Let X = {a, b, c}, IT_i = {X, ∅, <X, {a, c}, {b}>, <X, ∅, {a, b}>} and IT_j = {X, ∅, <X, ∅, {b, c}>, <X, {a, c}, ∅>} be bi-ITS. Here, <X, {a}, ∅>, <X, {a}, {b}>, <X, {a}, {c}>, <X, {a, b}, ∅>, <X, {a}, {b, c}> and <X, {a, b}, {c}> are (i, j)-Ipre open set but not (i, j)-IP*O (X, IT_i, IT_j).

Theorem 3.16 In a bi-ITS (X, IT_i, IT_j), each (i, j)-Ipre*open set is (i, j)-Iβ open set.

Proof: Let $\Re$ be (i, j)-Ipre*open set then $\Re \subseteq IT_i int(IT_j cl^*(\Re)) \subseteq IT_j cl(IT_i int(IT_j cl(\Re)))$. Hence, $\Re$ is (i, j)-Iβ open set.

Remark 3.17 The afterwards example shows that the previous theorem converse doesn't have too accurate.

Example 3.18 Let X = {a, b, c}, IT_i = {X, ∅, <X, {b}, ∅>, <X, {b}, {c}>, <X, {b, c}, ∅>} and IT_j = {X, ∅, <X, {a}, {b}>, <X, ∅, {b}>, <X, {a}, ∅>, <X, {a, c}, ∅>}be a bi-ITS. Here, <X, {a}, ∅>, <X, {a}, {b}>, <X, {a}, {c}>, <X, {c}, {a}>, <X, {a, b}, ∅>, <X, {b, c}, {a}>, <X, {a}, {b, c}>, <X, {a, b}, {c}> and <X, {c}, {a, b}> are (i, j)-Iβ open set but not (i, j)-IP*O (X, IT_i, IT_j).

Theorem 3.19 Let (X, IT_i, IT_j) be bi-ITS, $\Re$ be an (i, j)-Ipre*open set and $\Im$ be an intuitionistic set such that $\Im \subseteq \Re \subseteq IT_j cl^*(\Im)$ then $\Im$ is also an (i, j)-Ipre*open set.

Proof: Suppose $\Re$ is an (i, j)-Ipre*open set then $\Re \subseteq IT_i int(IT_j cl^*(\Re))$. Since, $\Re \subseteq IT_j cl^*(\Im)$ then $IT_j cl^*(\Re) \subseteq IT_j cl^*(\Im)$. Also, $\Im \subseteq \Re \Rightarrow \Im \subseteq IT_i int(IT_j cl^*(\Im))$. Thus, $\Im$ is (i, j)-Ipre*open set.

Theorem 3.20 If $\Re$ is an (i, j)-Ipre*open set and $\Im$ is IT_i-open set then $\Re \cup \Im$ is (i, j)-Ipre*open.

Proof: proof is obvious by theorem 3.5 and 3.8.

Remark 3.21 The concept of (i, j)-Isemi open set and (i, j)-Ipre*open set are independent.

Example 3.22 Let X = {p, q}, IT_i = {X, ∅, <X, ∅, ∅>, <X, {p}, ∅>, <X, {q}, ∅>, <X, ∅, {q}>} and IT_j = {X, ∅, <X, ∅, {p}>, <X, {p}, ∅>, <X, {q}, {p}>}. Clearly that, <X, {q}, {p}> is (i, j)-IP*O (X, IT_i, IT_j) but not (i, j)-Isemi open set and <X, {p}, {q}> is (i, j)-Isemi open set but not (i, j)-IP*O (X, IT_i, IT_j).

Remark 3.23 The concept of (i, j)-Iα open set and (i, j)-Ipre*open set are independent.

Example 3.24 Let X = {s, t}, IT_i = {X, ∅, <X, ∅, ∅>, <X, {t}, ∅>, <X, ∅, {t}>} and IT_j = {X, ∅, <X, {t}, {s}>}. Here, (i, j)- IP*O (X)= {X, ∅, <X, ∅, ∅>, <X, {t}, ∅>, <X, ∅, {t}>, <X, ∅, {s}>, <X, {t}, {s}>} and (i, j)- IαO(X) = {X, ∅, <X, ∅, ∅>, <X, {t}, ∅>, <X, ∅, {t}>, <X, {s}, ∅>}. Clearly that, <X, ∅, {s}>, <X, {t}, {s}> are (i, j)-IP*O (X, IT_i, IT_j) but not (i, j)- Iα open set and <X, {s}, ∅> is (i, j)- Iα open set but not (i, j)- IP*O (X, IT_i, IT_j).

Remark 3.25 The concept (j, i)-IP*O (X, IT_i, IT_j) and (i, j)-IP*O (X, IT_i, IT_j) are independent.

Example 3.26 Consider example 3.2 & 3.4. <X, ∅, {a, c}> & <X, {a, c}, {b}> are (i, j)-IP*O (X, IT_i, IT_j) but not are (j, i)-IP*O (X, IT_i, IT_j) and <X, ∅, ∅>, <X, ∅, {b}>, <X, ∅, {c}>, <X, {c}, ∅> <X, {a}, {c}>, <X, ∅, {b, c}>, <X, {a, b}, {c}> & <X, {a}, {b, c}> are (j, i)-IP*O (X, IT_i, IT_j) but not (i, j)-IP*O (X, IT_i, IT_j).

Definition 3.27 Let (X, IT_i, IT_j) be bi-ITS, $\Re \subseteq X$, $\Re$ is said to be

a) (i, j)-intuitionistic semi*open set (shortly, (i, j)-IS*OS) if $\Re \subseteq (IT_j cl^*(IT_i int(\Re))$.

b) (i, j)-intuitionistic α^*open set (shortly, (i, j)-Iα^*OS) if $\Re \subseteq IT_i int^*(IT_j cl(IT_i int^*(\Re)))$.

c) (i, j)-intuitionistic β^*open set (shortly, (i, j)-Iβ^*OS) if $\Re \subseteq (IT_j cl(IT_i int^*(IT_j cl(\Re)))$.

Theorem 3.28 In a bi-ITS (X, IT_i, IT_j), each (i, j)-Ipre*open set is (i, j)-Iβ^*open set.

Proof: Let $\Re$ be an (i, j)-Ipre*open set in X, then $\Re \subseteq IT_i int(IT_j cl^*(\Re))$. By 3.13 we have, $\Re \subseteq IT_i int(IT_j cl^*(\Re)) \subseteq IT_i int(IT_j cl(\Re)) \subseteq IT_j cl(IT_i int^*(IT_j cl(\Re))) \Rightarrow \Re \subseteq (IT_j cl(IT_i int^*(IT_j cl(\Re)))$. Thus, $\Re$ is (i, j)-Iβ^* open set.

Remark 3.29 The afterwards example shows that the previous theorem converse doesn't have to be accurate.

Example 3.30 Let X = {1, 2, 3}, IT_i = {X, ∅, <X, {1, 2}, {3}>, <X, {1}, {2, 3}>, <X, {2}, {1, 3}>} and IT_j = {X, ∅, <X, {1}, {2}>, <X, {2}, {3}>, <X, {1, 2}, ∅>, <X, ∅, {2, 3}>} be bi-ITS. Here, <X, ∅, ∅>, <X, ∅, {1}>, <X, {1}, ∅>, <X, ∅, {2}>, <X, {2}, ∅>, <X, ∅, {3}>, <X, {3}, ∅>, <X, {1}, {2}>, <X, {2}, {1}>, <X, {2}, {3}>, <X, {3}, {2}>, <X, {1}, {3}>, <X, {3}, {1}>, <X, ∅, {1, 2}>, <X, ∅, {2, 3}>, <X, {2, 3}, ∅>, <X, ∅, {1, 3}>, <X, {1, 3}, ∅>, <X, {2, 3}, {1}>, <X, {1, 3}, {2}> and <X, {3}, {1, 2}> are (i, j)-Iβ^*open set but not (i, j)-IP*O (X, IT_i, IT_j).

Remark 3.31 The concept of (i, j)-Iα^*open set and (i, j)-Ipre*open set are independent.

Example 3.32 Let X = {r, k}, IT_i = {X, ∅, <X, ∅, ∅>, <X, {r}, ∅>, <X, {k}, ∅>, <X, ∅, {k}>} and IT_j = {X, ∅, <X, ∅, {r}>, <X, {r}, ∅>, <X, {k}, {r}>}. Clearly that, <X, {k}, {r}> is (i, j)-IP*O (X, IT_i, IT_j) but not (i, j)-Iα^*open set and <X, {r}, {k}> is (i, j)-Iα^*open set but not (i, j)-IP*O (X, IT_i, IT_j).

Remark 3.33 The concept of (i, j)-Isemi*open set & (i, j)-Ipre*open set are independent.

Example 3.34 Let X = {x, y, z}, IT_i = {X, ∅, <X, {x}, {y, z}>, <X, {z}, {x, y}>, <X, {x, z}, {y}>} and IT_j = {X, ∅, <X, {x}, {y}>, <X, {x, z}, ∅>}. Here, <X, {x, z}, ∅> is (i, j)-IP*O (X, IT_i, IT_j) but not (i, j)-Isemi*open set. <X, {z}, {x}>, <X, {x}, {z}>, <X, {y, z}, {x}> and <X, {x, y}, {z}> are (i, j)-Isemi*open set but not (i, j)-IP*O (X, IT_i, IT_j).

4. CONCLUSION

This work introduces & studies (i, j)-Ipre* open sets in bi-ITS has been introduced and investigated. Fundamental properties and several characterizations of (i, j)-Ipre*open sets were established with the aid of suitable examples. The introduced concept provides a useful framework for studying more generalized structures in bi-intuitionistic topology. Future research may focus on applications of (i, j)-Ipre*open sets in continuity, compactness, connectedness, separation axioms, and related generalized mappings in bi-ITS.

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